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Coupling Equivalent Plate and Finite Element Formulations in Multiple-Method Structural Analyses

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A coupled multiple-method analysis procedure for use late in conceptual design or early in preliminary design of aircraft structures is described. Using this method, aircraft wing structures are represented with equivalent plate models, and structural details such as engine/pylon structure, landing gear, or a "stick" model of a fuselage are represented with beam finite element models. These two analysis methods are implemented in an integrated multiple-method formulation that involves the assembly and solution of a combined set of linear equations. The corresponding solution vector contains coefficients of the polynomials that describe the deflection of the wing and also the components of translations and rotations at the joints of the beam members. Two alternative approaches for coupling the methods are investigated; one using transition finite elements and the other using Lagrange multipliers. The coupled formulation is applied to the static analysis and vibration analysis of a conceptual design model of a fighter aircraft. The results from the coupled method are compared with corresponding results from an analysis in which the entire model is composed of finite elements.

Nomenclature

A, B, C = polynomial coefficients for displacement functions in u, v, w directions
c = combination of A, B, C into a single vector, Eq. (4)
d = finite element displacement field
f = element applied loads vector
G = transformation matrix
I = identity matrix
K = stiffness matrix
k = element stiffness matrix
P = generalized applied loads vector
T = transformation matrix
U, V, W = equivalent plate displacement functions
X, Y = power series functions in x and y, respectively
x, y, z = Cartesian coordinates
β = transformation matrix
Δ = element displacement vector
δ = translations at ends of beam element
θ = rotations at ends of beam element
λ = Lagrange multiplier
Φ = camber magnitude

Subscripts
b = beam
c = constrained
eq = equivalent plate
f = unconstrained or free
int = interface
o = reference surface
p = a particular point
u, v, w = deformations in x, y, z directions
$,x$ = differentiation with respect to x
$,y$ = differentiation with respect to y
1, 2 = ends of a beam element

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Introduction

A VARIETY of analysis methods have been developed for use by the various engineering disciplines involved in the design and analysis of advanced aerospace vehicles. Within each discipline, special purpose analysis methods exist for obtaining results with different levels of detail and accuracy. These methods range from rapid/approximate methods for assessing global behavior during early preliminary design to methods for accurate analysis of local details during final design. Historically, the methods have been used separately to produce needed analysis and design data. In recent years, efforts to provide improved designs have pointed to the following needs: 1) to include the effects of multidisciplinary coupling, 2) to increase the level of detail of the design data associated within each discipline, and 3) to introduce both of these improvements as early as possible in the design process in order to reduce the design cycle time and at the least possible cost to permit the repeated analyses required in the design environment.

Selected analysis methods have been combined in multiple-method formulations as one approach to help meet these needs. In Ref. 1, multiple-method analyses are identified according to the type of interface existing between the methods, and some concepts for integrating these methods are presented. An example of applying analysis methods from different disciplines is a boundary condition coupling procedure associated with calculating a consistent set of structural deflections and aerodynamic pressure loads on the external surface (the "boundary" between the two methods) of an aircraft wing using an iterative aeroelastic procedure.[2] The terminology, coupled-field problems, is used to describe another example of multiple-method formulations where the analysis methods associated with different disciplines are each applied over the same region or field. Partitioned analysis procedures for analysis of coupled-field dynamical problems such as thermal-structural interactions using a staggered solution procedure are described in Ref. 3. Distinct from multiple-methods, but closely related, is a class of analysis methods characterized by the use of a single disciplinary analysis method that is applied with different levels of modeling refinement in different regions of an analytical model. Such methods are often used to compute accurate results in local areas with high gradients of response quantities. Examples of these formulations include substructuring,[4] multigrid,[5] and global-local[6-10] analysis methods.

In this article, different analysis methods from the same discipline (structures) are combined in a single formulation. In general, for aircraft applications, the wing structure could be modeled as equivalent plates and the fuselage would be represented by conventional finite elements. However, p-method finite elements or boundary elements might also be used for an accurate analysis of a local detail. Such a coupled multiple-method formulation enables the analyst to develop a single analytical model, taking advantage of the particular strengths of different methods, to perform a particular task. The coupled multiple-method analysis procedure developed herein is intended for use in design studies of aircraft structures during late conceptual design and/or early preliminary design.

Simplified beam or plate models are often used for rapid/approximate analysis of aircraft wing structures during early preliminary design.[11] For example, an equivalent plate model of the wing structure is used in the aeroelastic tailoring and structural optimization (TSO) computer program, which has had widespread use for aeroelastic tailoring of composite wings.[12-14] Recently, a new equivalent plate formulation with a more generalized structural modeling capability has been developed and implemented in a computer program equivalent laminated plate solution (ELAPS).[15,16] This approach has been useful for multidisciplinary analysis studies.[17] However, the structural models are constructed entirely from plate segments and spring members, and there is a continuing need to further generalize the modeling capabilities for representation of additional details during aircraft structural design.

This article describes the addition of beam finite elements to the equivalent plate modeling capability. These beam members can be used to model structural details such as engine/pylon structure, landing gear, or provide a stick representation of a fuselage. Such a modeling capability provides the necessary detail to represent the dynamic characteristics of an entire vehicle structure in a flutter analysis or optimization procedure during early preliminary design. A brief description of each analysis method is presented followed by a more detailed description of the interface procedures used to couple the analysis methods. Two alternative approaches for coupling the methods are investigated: 1) one using transition finite elements and 2) the other using Lagrange multipliers. The resulting coupled multiple-method analysis procedures are used to perform the structural analysis of a conceptual design model representative of a fighter aircraft in which the wing structures are represented by equivalent plates and the fuselage and wing store/pylon structure are modeled using beam finite elements. The results from the coupled multiple-method analyses are compared with results from a model composed entirely of finite elements.

Structural Analysis Methods

The two different structural analysis methods that are combined into a single multiple-method technique are indicated schematically in Fig. 1, where the wing structure is modeled as equivalent plates and the fuselage is represented by conventional finite elements. The integrated multiple-method approach used herein is formulated as a partitioned analysis problem as shown in Eq. (1). Only two partitions are used: 1) one for the equivalent plate method and 2) one for conventional beam finite elements, written in schematic form as

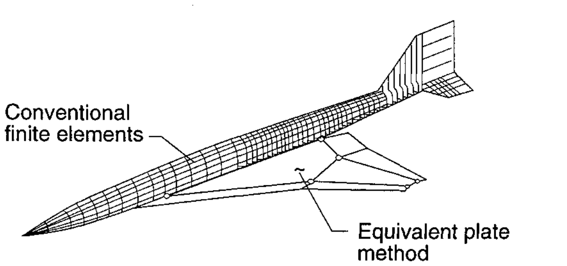

Fig. 1 Multiple-method approach for analysis of aircraft structures.

$$\begin{bmatrix} K_{eq} & K_{int} \\ K_{int}^T & K_b \end{bmatrix} \begin{Bmatrix} \text{polynomial coefficients} \\ \text{joint displacements} \end{Bmatrix} = \begin{Bmatrix} \text{generalized forces} \\ \text{joint forces} \end{Bmatrix} \tag{1}$$

Only a brief review of each method is presented herein along with a more detailed description of the interface procedure used to combine the methods. The reader is directed to reference material for a detailed description of procedures used to generate the governing equations for these two analysis methods.

Equivalent Plate Method

The Ritz method is based on minimizing the energy of the structure and applied loads. As in Refs. 15 and 16, each component of the deformation of the reference surface of the equivalent plate is assumed to be the sum of contributions from sets of specified displacement functions

$$U_o = \sum A_i X(x)_i Y(y)_i \tag{2a}$$

$$V_o = \sum B_j X(x)_j Y(y)_j \tag{2b}$$

$$W_o = \sum C_k X(x)_k Y(y)_k \tag{2c}$$

where U_o and V_o refer to stretching of the reference surface in the x and y directions at $z = 0$, and W_o is the bending deformation normal to the surface. The displacement functions are specified as products of terms from a power series in the x direction with terms from a power series in the y direction. Different sets of functions can be specified by the analyst for the U_o, V_o, and W_o components of the deformation. In this study, five terms of a power series are used in the x direction and six terms of a power series are used in the y direction for all three components of deformation, (i.e., $i = j = k = 5$ for $X(x)$ and $i = j = k = 6$ for $Y(y)$.

For static analysis, the Ritz procedure produces a system of simultaneous linear equations that can be solved for the unknown coefficients in Eq. (2) to minimize the total energy expression. These equations are written in partitioned matrix form as

$$\begin{bmatrix} K_{uu} & K_{uv} & K_{uw} \\ K_{vu} & K_{vv} & K_{vw} \\ K_{wu} & K_{wv} & K_{ww} \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{P}_u \\ \boldsymbol{P}_v \\ \boldsymbol{P}_w \end{Bmatrix} \tag{3}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ are vectors of the associated polynomial coefficients in Eq. (2). These equations represent the fully coupled bending-stretching behavior of a composite plate. The procedure used to evaluate the terms in the submatrices of the stiffness matrix and load vector of Eq. (3) is described in Ref. 16. These equations are written in a compressed form for subsequent use as

$$K_{eq}\boldsymbol{c}_{eq} = \boldsymbol{P}_{eq} \tag{4}$$

where K_{eq} is the stiffness matrix, P_{eq} are generalized applied loads, and c_{eq} are wing deformation coefficients expressed in terms of A, B, and C.

Beam Finite Elements

In the conventional finite element approach for a static analysis, the linear set of governing equations have the form

$$K_b\boldsymbol{d}_b = \boldsymbol{P}_b \tag{5}$$

The global stiffness matrix K_b is assembled from contributions of each of the individual elemental stiffness matrices, $\boldsymbol{P}_b$ is a vector of forces and moments at the ends of the beams, and $\boldsymbol{d}_b$ is a vector of translations and rotations at joints on the ends of the beams. The elemental beam stiffness matrices in local coordinates and the matrices used for transformation of these elements from the local to global coordinate systems are given in Ref. 18. The equations for each beam element in global coordinates can be written as

$$k_b \boldsymbol{\Delta}_b = f_b \tag{6}$$

These elemental stiffness matrices k_b are of the order 12 × 12, corresponding to three translations and three rotations at each end of a beam.

Interface Procedures

A schematic for a portion of an analytical model using an equivalent plate model for the wing structure and a beam finite element model for a wing store/pylon structure is shown in Fig. 2. The interface procedure used herein is to equate the values of deflections of the equivalent plate to the values of the deflections at the ends of the beam elements at the interface joints shown by the darkened symbols in Fig. 2b.

The deformations in the equivalent plate are given as

$$U = U_o - zW_{o,x} \tag{7a}$$

$$V = V_o - zW_{o,y} \tag{7b}$$

$$W = W_o \tag{7c}$$

These deformations can be expressed in terms of the polynomial coefficients A, B, and C at a particular point p, by substituting Eq. (2) into Eq. (7) and evaluating the expressions at the point (x_p, y_p, z_p), resulting in the form

$$\begin{Bmatrix} U \\ V \\ W \end{Bmatrix}_p = T_p \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = T_p \boldsymbol{c}_{\text{eq}} \tag{8}$$

where T_p is a transformation matrix. At a point p, the deformations of the equivalent plate (U, V, and W) given in Eq. (8) are equated to the translations in the x, y, and z directions at the end of a beam $(\delta_x, \delta_y, \delta_z)$, to give

$$\begin{Bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{Bmatrix}_p = T_p \boldsymbol{c}_{\text{eq}} \tag{9}$$

The relationship in Eq. (9), between the equivalent plate coefficients and the beam joint displacements at all points where the beams are connected to the plates, is used in both interface procedures that follow.

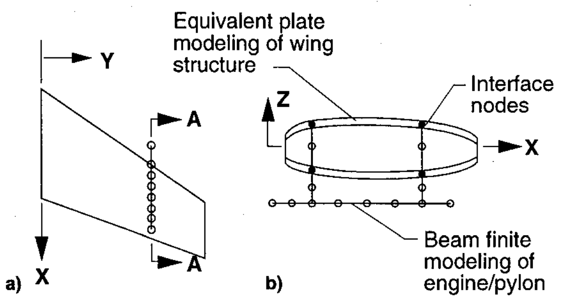

Fig. 2 Schematic of multiple-method modeling: a) wing planform and b) section A-A.

Transition Finite Elements

A transition finite element refers to a beam element in which the three translations δ_1 at end 1 are expressed in terms of the plate coefficients. The joint rotations θ_1 at end 1 are assumed to be independent of the plate coefficients, and hence form a "pinned" connection with the plate. The three translations δ_2, and three rotations θ_2 at end 2 are conventional beam joint degrees of freedom (DOF). Using Eq. (9), the displacement vector $\boldsymbol{\Delta}_b$, corresponding to the beam elemental stiffness matrix given in Eq. (6) can be transformed into a combination of the polynomial coefficients and translations and rotations of the beam joints as

$$\begin{Bmatrix} \delta_1 \\ \theta_1 \\ \delta_2 \\ \theta_2 \end{Bmatrix} = \begin{bmatrix} T_p & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{Bmatrix} \boldsymbol{c}_{\text{eq}} \\ \theta_1 \\ \delta_2 \\ \theta_2 \end{Bmatrix} \tag{10}$$

Defining the transformation matrix in Eq. (10) as β and performing the operation

$$k_t = \beta^T k_b \beta \tag{11}$$

yields a transition beam stiffness matrix k_t that is used to attach conventional beam finite elements to the equivalent plate structure. When assembled into the global stiffness matrix of Eq. (1), the transition beam stiffness matrix adds terms to the submatrices denoted K_{eq} and K_b, and forms the interface portion K_{int}. This assembly process required a detailed understanding of the data structure used to store K_{eq} and K_b. The same interface procedure could be used to attach other types of finite elements, such as membrane and bending plate elements, to the equivalent plate model. Such a capability would allow analysis of a fuselage that is built up of elements representing rings, stringers, and skin panels connected to the equivalent plate model. For this approach, transition elements for all types of finite elements that could be attached to the equivalent plate would have to be formulated.

Lagrange Multipliers

An alternate formulation, using Lagrange multipliers,[19] leaves K_{eq} and K_b undisturbed. This alternate approach offers the advantage of not requiring a detailed understanding of the element stiffness matrices or associated data structures. In this formulation, the stiffness matrices for the equivalent plate and beam methods are assembled independently without any consideration of coupling. These matrices are then coupled using the Lagrange multiplier approach. The coupling is accomplished using constraint equations, as given in Eq. (9), to equate the displacement components at the ends of all the attached beams to corresponding deformations on the equivalent plate as

$$\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \vdots \\ \delta_N \end{Bmatrix} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ \vdots \\ T_N \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} \tag{12}$$

where N is the number of attachment points between the beams and the equivalent plate. This set of constraint equations can be written as

$$G\boldsymbol{c}_{\text{eq}} - \boldsymbol{d}_c = 0 \tag{13}$$

where

G = transformation matrix
c_{eq} = vector of deformation coefficients for the equivalent plate

d_c = vector of beam deformations that are coupled to the plate

When using the Lagrange multiplier approach[20] to apply the constraints given in Eq. (13), each of the constraint equations is multiplied by a Lagrange multiplier λ_i, and the product, by which the definition is equal to zero [see Eq. (13)], is used to augment the energy expression for the entire system as

$$\Pi = \frac{1}{2} c_{eq}^T K_{eq} c_{eq} + \frac{1}{2} d_b^T K_b d_b - P_{eq}^T c_{eq} - P_b^T d_b + \lambda^T (G c_{eq} - d_c) \quad (14)$$

The beam displacements d_b can be partitioned into d_c translational DOF that are connected to the equivalent plate by interface constraints, and d_f, the remaining unconstrained translational and rotational DOF. Taking the variation of the energy expression given in Eq. (14) with respect to c_{eq}, d_f, d_c, and λ and equating the result to zero gives the following governing equations:

$$\begin{bmatrix} K_{eq} & 0 & 0 & G^T \\ 0 & K_{ff} & K_{fc} & 0 \\ 0 & K_{fc}^T & K_{cc} & -I \\ G & 0 & -I & 0 \end{bmatrix} \begin{Bmatrix} c_{eq} \\ d_f \\ d_c \\ \lambda \end{Bmatrix} = \begin{Bmatrix} P_{eq} \\ P_f \\ P_c \\ 0 \end{Bmatrix} \quad (15)$$

This approach has an advantage in that the stiffness matrices for the equivalent plates and beams are unaltered from their original form. Hence, a detailed understanding of the procedures used to generate these matrices is not required in order to use them in a coupled formulation. In fact, these matrices could be assembled in two completely separate computer programs and then transferred to a third program that would form and solve the coupled equations in Eq. (15).

The disadvantage of the coupled formulation in Eq. (15) is that it produces a larger system of equations than the transition element approach. In addition, the direct solution of Eq. (15) requires the use of a solver that can handle the zero terms that are produced on the diagonal of the stiffness matrix. In this study, Eq. (15) is not solved directly. Instead, Eq. (15) is manipulated to provide insight into the Lagrange multiplier approach. Using the last three sets of partitioned equations, d_f, d_c, and λ are expressed in terms of c_{eq}, and these expressions are substituted into the first set of equations to give

$$[K_{eq} + G^T(K_{cc} - K_{fc}^T K_{ff}^{-1} K_{fc})G] c_{eq} = \{P_{eq} - G^T(-P_c + K_{fc}^T K_{ff}^{-1} P_f)\} \quad (16)$$

The vector $(-P_c + K_{fc}^T K_{ff}^{-1} P_f)$ and the matrix $(K_{cc} - K_{fc}^T K_{ff}^{-1} K_{fc})$ can be obtained using a finite element analysis program in which applied displacements are specified as input quantities and the remaining displacements along with the joint reaction forces are calculated using the following set of equations:

$$\begin{bmatrix} K_{ff} & K_{fc} \\ K_{fc}^T & K_{cc} \end{bmatrix} \begin{Bmatrix} d_f \\ d_c \end{Bmatrix} = \begin{Bmatrix} P_f \\ P_c \end{Bmatrix} + \begin{Bmatrix} 0 \\ \lambda \end{Bmatrix} \quad (17)$$

where λ are the unknown forces which would be required to produce the specified displacements d_c.

Specifically, if the applied displacements d_c are given, then the unknown quantities, d_f and λ are given by

$$d_f = K_{ff}^{-1}(P_f - K_{fc} d_c) \quad (18)$$

$$\lambda = (-P_c + K_{fc}^T K_{ff}^{-1} P_f) + (K_{cc} - K_{fc}^T K_{ff}^{-1} K_{fc}) d_c \quad (19)$$

The vector and matrix needed in Eq. (16) are contained in Eq. (19) and can be formed using the following procedure in the finite element program:

1) To form $(-P_c + K_{fc}^T K_{ff}^{-1} P_f)$: Calculate the reaction force vector λ when the forces P_f and P_c are applied, and with all of the DOF associated with d_c, set equal to zero.

2) To form $(K_{cc} - K_{fc}^T K_{ff}^{-1} K_{fc})$: Calculate the matrix of reaction force vectors with $P_c = 0$, $P_f = 0$, and $d_c = I$. These vectors are produced using a set of applied displacements in which unit displacements are systematically applied to each of the DOF in d_c with the other DOF set equal to zero.

This solution feature is available in most general-purpose finite-element programs. The above quantities are the so-called reduced load vector and stiffness matrix for the beam structure in terms of the interface DOF. These quantities may be formed for the beam finite element portion of the model using an existing finite element program and transferred to the equivalent plate analysis program for use in the solution of Eq. (16).

Comparison of Coupling Methods

The two interface methods considered in this study have been discussed in the literature[18,19] and are both used in this study to assess which method is more effective to couple analyses that are intended for use in an iterative design environment. In both methods, the deformations match exactly where the equivalent plate and finite element models are joined. For the transition finite element method, this matching is performed on an individual finite element basis using the transformation matrix β in Eq. (11). For the Lagrange multiplier method, the matching is performed using the transformation matrix G, which corresponds to all finite element joints that are connected to the equivalent plate model. Although the two methods result in different sets of governing equations, they are theoretically identical and are shown to produce identical results in this study. Thus, the comparison of the methods is focused on the relative ease of implementation and computational efficiency for generating and solving the resulting sets of governing equations.

The Lagrange multiplier method has a significant advantage over the transition element method for use in design studies. When Lagrange multipliers are used the stiffness matrix for the equivalent plates and the stiffness matrix for the finite element model are kept separate and are unaltered from their original form in the coupled formulation. Once the reduced load vector $(-P_c + K_{fc}^T K_{ff}^{-1} P_f)$ and the reduced stiffness matrix $(K_{cc} - K_{fc}^T K_{ff}^{-1} K_{fc})$ shown in Eq. (16) are computed for the finite element model, many wing designs can be studied by changing design variables that alter the stiffness matrix of the equivalent plate K_{eq} without regenerating the information from the finite element model. In addition, the reduced load vector and reduced stiffness matrix for the finite element model can be easily formed using standard features that are available in most general-purpose finite element programs. These procedures involve calculating reaction force vectors for specified values of displacements at each of the interface DOF. As described in the preceding section, these DOF are set equal to zero to calculate the reduced load vector, and a unit displacement or motion is systematically applied to each one in order to calculate the reduced stiffness matrix.

The transition element method has the disadvantage that terms from the transition finite element stiffness matrix must be added to terms in the stiffness matrix for the equivalent plate and the stiffness matrix of the remainder of the finite element model. This assembly process requires a detailed understanding of the data structures used to store these matrices and the process is more difficult to implement in a computer program. Moreover, a separate interface element would have to be formulated for each additional type of finite element (e.g., membrane and bending plates, shear panels, and brick elements) to extend the procedure beyond the beam element used in this study.

Methods Implementation

Transition Elements

In this approach, procedures for data input and generation/assembly of stiffness matrices associated with the beam elements were added to the ELAPS program. In this initial demonstration of a coupled multiple-method approach, there was no attempt to optimize the computational efficiency by taking advantage of the sparse nature of the assembled beam stiffness matrix. The entire upper triangle of the symmetric global stiffness matrix was assembled. However, the two different types of unknown quantities, i.e., polynomial coefficients and joint displacements, required that a correspondence table be used in assembling the global stiffness matrix.

Correspondence tables were constructed that assigned a sequential degree-of-freedom number to each of the polynomial coefficients and the three translations and three rotations at each beam joint. Using these correspondence tables, vectors that map each row and column of the transition beam stiffness matrix to generalized DOF were formed and used to assemble these matrices into the proper locations in the global stiffness matrix.

Lagrange Multipliers

In this approach, the reduced load and stiffness matrices were formed using the engineering analysis language (EAL)[21] finite element structural analysis program. These matrices were then transferred from EAL to the ELAPS program. In ELAPS, the transformation matrix $[G]$ was generated and used with the quantities from EAL to form Eq. (16). The unknown coefficients for the equivalent plate c_{eq} were calculated in ELAPS along with the deflections and stresses at a grid of points over the planform of the wing. The deflections at the interface constraints $\boldsymbol{d}_c$ were also calculated in ELAPS using Eq. (13) and were transferred from ELAPS to EAL to be used as applied displacements in the calculation of the remaining translations and rotations $\boldsymbol{d}_f$ and the corresponding stresses in the beam elements. Since the calculations are performed at the assembled stiffness matrix level, the Lagrange multiplier approach provides the analyst with the versatility to use a general finite element model containing additional types of elements (e.g., membrane and bending plates, shear panels, and brick elements) instead of being limited to the finite element beam model.

Application and Results

The coupled analysis procedure developed herein is used to perform the structural analysis of a conceptual design model representative of a fighter aircraft in which the wing structures are modeled by equivalent plates and the fuselage and wing store/pylon structure are modeled using beam finite elements. The equivalent plate model of the wing alone (without the beam elements) was studied in Ref. 16. The results from the coupled analysis, described herein, are compared with corresponding results from a model composed entirely of finite elements.

Model Description

A schematic of the model used to demonstrate the multiple-method analysis approach is shown in Fig. 3. A planform view is shown along with views at two vertical sections: 1) at the wing-fuselage intersection ($y = 40$), and 2) at the wing store/pylon location ($y = 140$). The wing structure is composed of a clipped-delta outer segment with a 45-deg leading-edge sweep and an inner segment to represent a carry-through structure. The wing cross section has a cambered shape. The midcamber surface is specified as a quadratic function of the chord length (parabolic arc) with its maximum out-of-plane dimension located at the 50% chord location. The amount of camber is defined as Φ, where Φ is the maximum out-of-plane dimension nondimensionalized by the local chord. Results are presented

Table 1 Properties of beam cross sections

Member	I_1, in.^{4}	I_2, in.^{4}	A, in.^{2}	J, lb-in.^{2}
Fuselage beam	7,200	28,800	8	57,600
Fuselage connection	0.4	0.4	0.8	0.8
Store/pylon beam	2.5	2.5	1.25	5.0
Store/pylon connection	0.1325	0.1325	0.471	0.265

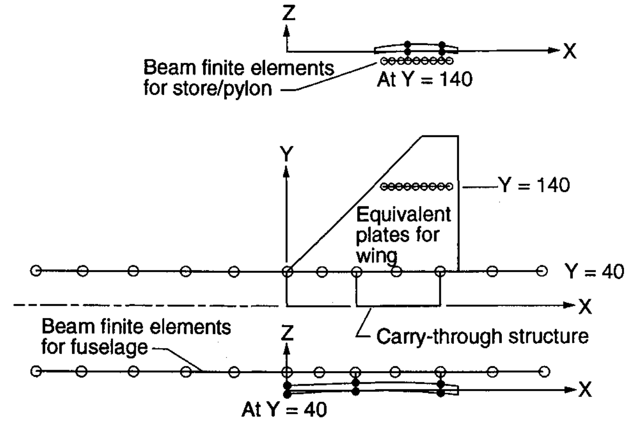

Fig. 3 Structural model for coupled multiple-method analysis.

in this study for a camber of $\Phi = 0.03$, which is the cambered configuration presented in Ref. 16.

The fuselage is represented as a beam located at the wing-fuselage intersection semispan location, rather than at the centerline of the vehicle. This location facilitates the connection of the wing to the fuselage and adequately represents the fuselage structure for deformations that are symmetric about the midplane of the vehicle. The fuselage beams are located at a height above the wing that is representative of the location of the centerline of a typical fuselage. The fuselage is connected to the wing at the wing leading edge and at the front and rear of the carry-through structure. At each connection point, one beam element is positioned between the upper and lower skins of the wing, and a second beam is located between the upper skin and the fuselage model. The wing store/pylon structure, located beneath the wing, is represented with a model that is similar to that used for the fuselage.

The joints at the ends of the beam elements are indicated by the circular symbols in Fig. 3. The stiffness properties of the beam elements are given in Table 1. All material properties are for aluminum. Concentrated masses are located at each of the joints along the fuselage and wing store/pylon. Magnitudes of the masses and mass moments of inertia, given in units of weight, are 100.0 and 1000.0 for each joint on the fuselage, and 5.0 and 20.0 for each joint on the wing store/pylon.

For static analysis, rigid-body motion of the structural model must be constrained. A combination of excluding selected terms from the plate displacement functions and equating to zero selected DOF at beam joints by removing the corresponding rows and columns from the stiffness matrix is used to define the constraints on the example model. Symmetric deflections about the midplane of the vehicle are imposed by omitting terms containing y^0 from the V deformation to provide a deflection constraint at $y = 0$, and by omitting terms containing y from the W deformation to constrain the slope in the spanwise direction at $y = 0$. For static analysis, the x and z DOF of the fuselage joint at the front of the carry-through structure are constrained, as well as the z DOF of the fuselage joint at the rear of the carry-through structure. For vibration analysis, these joint DOF are not constrained, and the vibration results include three rigid-body modes. An eigenproblem shift parameter is used to perform a vibration analysis on a model with unconstrained (rigid-body) motions.

There was concern that the set of equations from the coupled multiple-method formulation might be severely ill-conditioned since two different types of unknown quantities are involved. However, such ill-conditioning did not pose a problem for the example application since both static and dynamic analyses were performed successfully.

Results from the multiple method analysis are compared with corresponding results from the engineering analysis language (EAL) finite-element analysis program.[21] The planform layout of the finite element model defined for the EAL analysis is shown in Fig. 4. This model is built up of membrane elements to model the upper and lower cover skins and shear elements to model the rib and spar webs. The details of the arrangement of elements through the depth of a typical segment of the wing are illustrated in Fig. 4. The grid of cover skin elements shown in Fig. 4 has the same layout for the upper and lower surfaces of the wing. Shear webs are positioned vertically to connect the upper and lower wing surfaces. These webs connect all edges of the membrane elements in the upper cover skin to corresponding edges of the elements in the lower cover skin. Each joint where the elements are connected has three DOF (translations in the x, y, and z directions). The z coordinate of the cover skin joints are defined to correspond to the $\Phi = 0.03$ camber shape used to describe the equivalent plate model. Symmetric deflection about the midplane of the vehicle are imposed by constraining the y DOF of all midplane joints. The dark lines at $y = 40$ and $y = 140$ in Fig. 4 indicate the location of the beam elements that are used to model the fuselage and store/pylon structure. For static analysis, the same fuselage joints that were constrained on the multiple-method model are also constrained on this model composed of all finite elements. The constrained finite element model has 1565 DOF. For vibration analysis, these fuselage joints are not constrained and the vibration results include three rigid-body modes.

Numerical Results

The coupled multiple-method analysis procedure is used to analyze the model shown in Fig. 3. The transition element approach is used for both static analysis and vibration analysis. The Lagrange multiplier approach is used only for static analysis and produces results that are identical to those from the transition element approach. It could be extended to include dynamics, but is beyond the scope of this article since it requires augmenting the energy expression. Typical results are presented to demonstrate that the multiple-method formulations adequately represent the structural response characteristics of the vehicle.

For testing of the static analysis, a single load of 1000 lb acting in the vertical direction at the front of the wing store

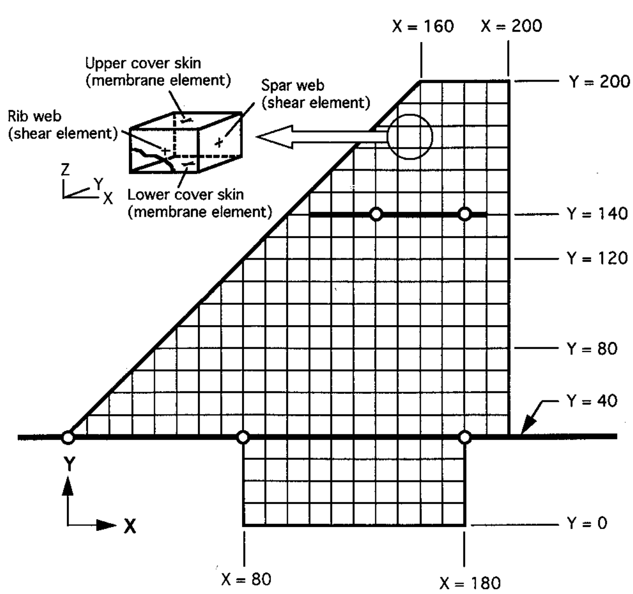

Fig. 4 Finite element model of wing structure.

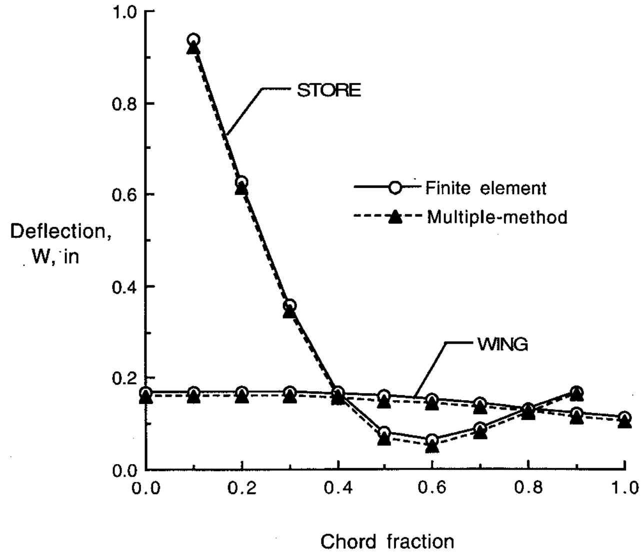

Fig. 5 Static deflection at $y = 140$.

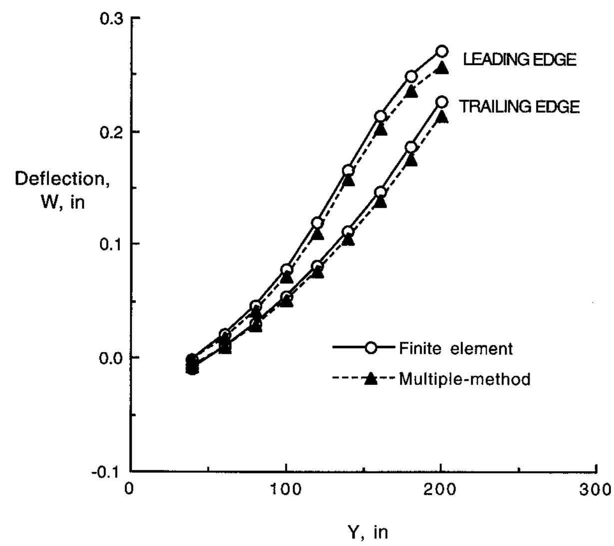

Fig. 6 Deflections of wing leading and trailing edges.

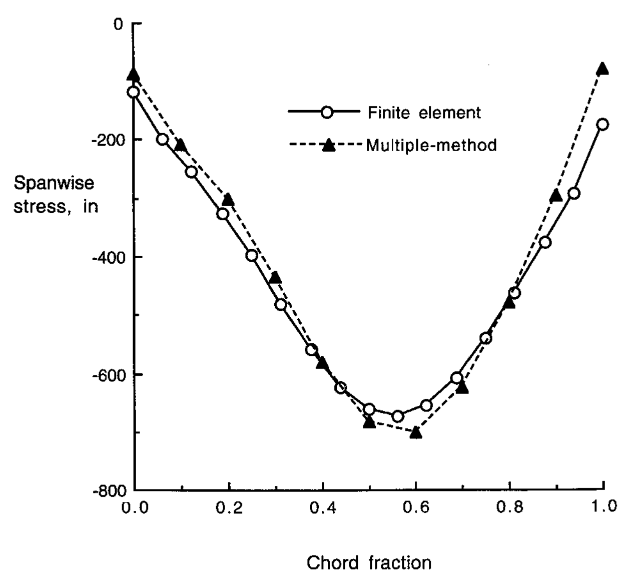

Fig. 7 Stress distribution at $y = 80$, upper cover.

Table 2 Comparison of natural frequencies from vibration analysis

Mode	Frequency, Hz Multiple-method	Finite element	% difference
1, 2, 3	0.00	0.00	0.0
4	5.27	5.09	4.3
5	11.25	10.99	2.4
6	17.36	16.94	2.5
7	25.83	25.84	0.0
8	29.00	29.00	0.0
9	30.84	30.84	0.0
10	33.35	33.29	0.2
11	34.90	34.90	0.0
12	35.88	35.17	2.0
13	39.33	38.98	0.9
14	43.27	43.15	0.3
15	53.63	54.16	1.0

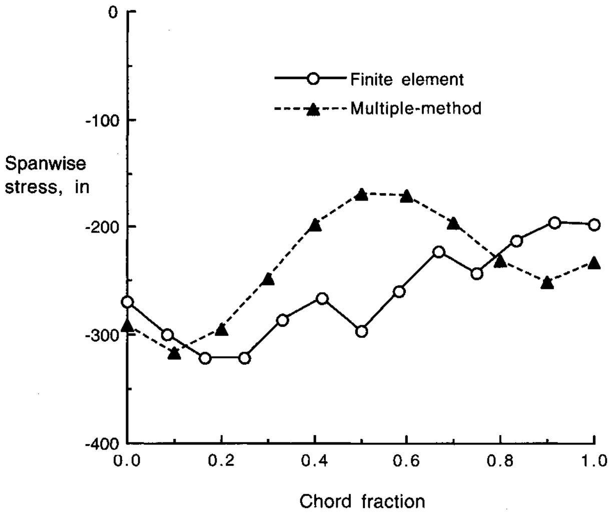

Fig. 8 Stress distribution at $y = 120$, upper cover.

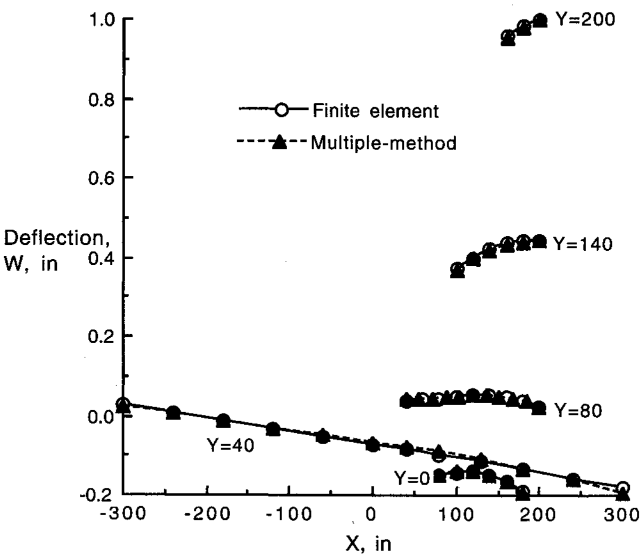

Fig. 9 Sixth vibration mode shape.

beam is used. The resulting deflection of the wing store beam and the deflection of the wing structure along the corresponding wing chord are shown in Fig. 5. The deflections of the wing and the store beam are the same at the attachment points for the 0.4- and 0.8-chord fraction locations. In addition, deflections along the leading and trailing edges of the wing are presented in Fig. 6. The static deflections for the multiple-method model are in close agreement with results for the model composed entirely of finite elements.

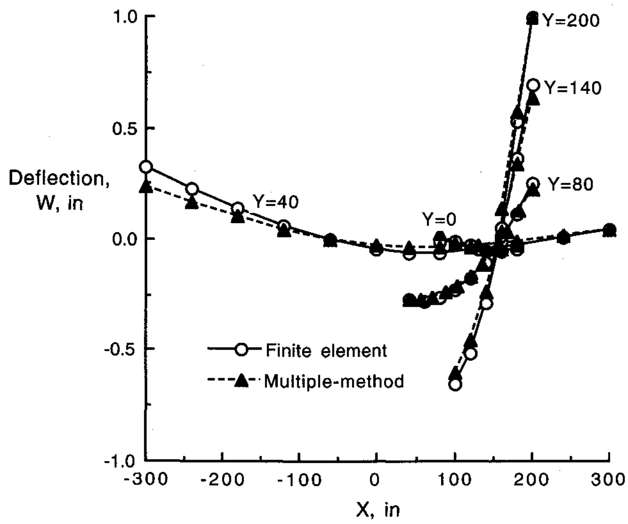

Fig. 10 Twelfth vibration mode shape.

Distributions of stress in the spanwise direction for the upper surface of the wing at two different semispan locations, $y = 80$ and $y = 120$, are shown in Figs. 7 and 8. The agreement in stresses between the multiple-method model and the finite element model is better at the inboard location ($y = 80$) of the wing than near the wing store/pylon ($y = 120$). The wing store/pylon structure transmits concentrated forces to the equivalent plate at the connection points. The resulting stress concentrations are not accurately predicted by the displacement functions assumed in the equivalent plate approach. These displacement functions are formulated to represent the global response of the wing and, hence, do not give adequate stress representation in areas of load introduction. In general, the stresses produced by the multiple-method analysis are of acceptable accuracy for use during preliminary design except in areas of high stress concentrations.

Vibration frequencies calculated for the symmetric model are presented in Table 2. The first three frequencies correspond to rigid-body modes in pitch, plunge, and fore/aft translation of the vehicle. The 12 other frequencies from the multiple-method approach, corresponding to flexible modes, are in good agreement with the results from the finite element analysis. The modal displacements at selected semispan locations are shown in Fig. 9 for the sixth vibration mode. This mode shape is dominated by bending of the wing and agreement between the multiple-method and finite element analysis is excellent. The 12th vibration mode, shown in Fig. 10, exhibits bending of the fuselage and torsion of the wing structure. Again, there is good agreement between the two analysis models. These vibration results indicate that the multiple-method analysis procedure provides an acceptable representation of the vibration characteristics of a vehicle for use during preliminary design.

Concluding Remarks

The implementation of a coupled multiple-method analysis procedure is demonstrated by combining finite element and equivalent plate analyses in a single formulation. The procedure is used to analyze the airframe of a fighter aircraft in which the wing is modeled using equivalent plates and the fuselage and wing store/pylon structures are modeled with beam finite elements. The set of governing equations contain coefficients of polynomials used to describe the deflection of the wing and the components of displacements and rotations of the joints of the beam elements. Potential ill-conditioning of the equations that might have been expected because of mixing of dissimilar unknowns, did not occur, since for both static and vibration analyses satisfactory solutions were obtained without the need for any scaling or special manipulation of the equations associated with either of the methods.

Two alternative approaches for coupling the methods are implemented: 1) one using transition finite elements, and 2) the other using Lagrange multipliers. Both approaches are used for static analysis and the results are identical. In the transition element approach, procedures for data input and generation/assembly of the stiffness matrices associated with both conventional and transition beam elements are added to the equivalent plate analysis program. The Lagrange multiplier approach is easier to implement since the stiffness matrices for both methods are generated and assembled in their original, unmodified form using separate computer programs, and subsequently coupled using constraint equations. This approach provides the analyst with the versatility to use a general finite element model containing additional types of elements (e.g., membrane and bending plates, shear panels, and brick elements) instead of being limited to the finite element beam model. The disadvantages of the Lagrange multiplier approach are that it yields a larger system of equations than the transition approach, and it requires a matrix solver that can handle a null partition on the main diagonal.

The combination of beam finite elements and equivalent plates provides a capability for creating a simplified model of a complete airframe. Results from both static and vibration analyses indicate that the coupled multiple-method analysis procedure provides an acceptable level of accuracy to represent the global structural response characteristics of a vehicle during early preliminary design.

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